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Star Identification Algorithm Based on Log-Polar Transform

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A star identification algorithm based on Log-Polar transform is proposed in this paper. First, Log-Polar transform is introduced to generate star patterns. Second, the generated star patterns are coded to strings for the purpose of quick match and memory saving. Finally, a modified string match algorithm based on Knuth–Morris–Pratt is used to find the matches between measured stars and guide stars. In simulations, using the stars brighter than magnitude 6.0 and a 12 × 12 degree field of view, the algorithm obtain an identification rate of 98% from the statistics of 1000 random sensor orientations at a positional noise level of two pixel. Meanwhile, the memory usage of the algorithm is comparatively small.

I. Introduction

As a kind of highly precise and reliable attitude measurement component, star sensors are widely used in attitude and orbit control systems of many spacecrafts [1–4]. In order to establish the initial attitude, a star identification routine is needed to find the corresponding matches between measured stars in field of view (FOV) and guide stars from an onboard catalog [5–10]. Star identification is the most crucial step for attitude determination.

Generally, existing star identification algorithms can be roughly divided into two classes [9]. The first class tends to approach star identification as an instance of subgraph isomorphism and uses star pair distance or triples to build guide database. These algorithms include triangle algorithm, polygon match algorithm, and group match algorithm. The second class assigns a unique pattern (star pattern) for each star and approach star identification in terms of pattern recognition. Star pattern-based algorithms show the advantage of robustness in finding the correct matches from incomplete or fragmented star patterns and become the study focus of star identification in recently years. However, how to build the unique star patterns effectively is a primary step and a kind of key technique for these algorithms.

Schwartz [11] has suggested that there is a Log-Polar mapping between human retina and eye-cortex and it plays an important role in scale, shift and rotation invariant target recognition. Log-Polar transform (LPT) is a kind of transform from Cartesian coordinates to polar coordinates of logarithmic radius. The scale, shift and rotation turn into single displacements through mapping, so the problem can be greatly simplified. LPT is widely used in many areas such as moving target recognition and optical character recognition [12,13].

A star identification algorithm based on LPT is presented in this paper by importing LPT to solve star identification problem. Star patterns are generated by LPT and then coded into strings, finally a string match algorithm is applied to find the matches between measured stars and guide stars.

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II. LPT of Star Image

Denoting original binary image in Cartesian coordinates and its LPT result image in polar coordinates as $f(x, y)$ and $g(r, \theta)$, respectively (as shown in Fig. 1), LPT from $f(x, y)$ to $g(r, \theta)$ can be defined as Eq. (1) as follows:

$$r = \ln\sqrt{x^2 + y^2}$$
$$\theta = \begin{cases} \tan^{-1}(y/x) & \text{if } x > 0,\ y > 0 \\ \pi + \tan^{-1}(y/x) & \text{if } y < 0 \\ 2\pi + \tan^{-1}(y/x) & \text{if } x < 0,\ y > 0 \end{cases} \tag{1}$$

With the help of LPT, the rotation and scale in original image turn into circular shift in θ-coordinate and shift in r-coordinate, respectively, in polar system. This kind of characteristic is the reason why LPT is usually chosen as a useful method to extract rotation and scale invariant features in image matching.

Considering the measured star image as rotation of one part of celestial sphere with the same size of FOV, star identification is equivalent to image matching in the condition of rotation. If a measured star image is rotated to coincide with one part of celestial sphere, a valid match comes out. Based on the above analysis, LPT can be employed to extract the rotation invariant features in star identification. Different from general cases, LPT for star identification needs only to make transform for discreet points (individual stars) instead of whole image, and centroids of stars are chosen as coordinate origins in LPT.

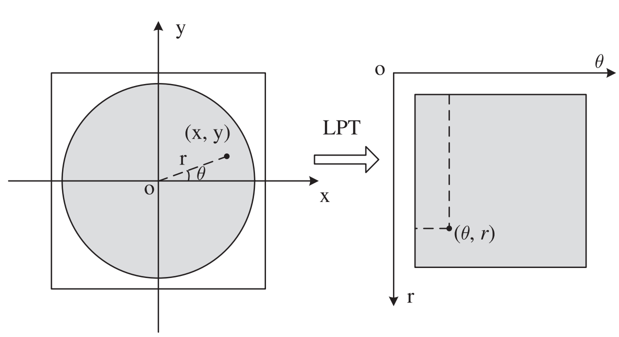
Generally, star patterns are generated from the geometric distribution of the neighboring stars. Assuming t is a star in either a star image or a guide star catalog, the star pattern of star t derives from the LPT of star image that consists of its neighboring stars. Without loss of generality, LPT of measures star image is used to illustrate the transform process of star image (as shown in Fig. 2). The steps of LPT are as follows: First, star image (star t and its neighboring stars within radius of R) is shifted to make sure star t is located at the origin. Second, the shifted star image (only coordinates of stars) is transformed by LPT according to Eq. (1) and then digitalized by $m \times n$ (m and n are sampling points in θ and r direction, respectively). The result image can be denoted as an $m \times n$ sparse matrix

$$A(i, j) = \begin{cases} 1 & \text{neighbor stars at } (i, j) \\ 0 & \text{no neighbor stars at } (i, j) \end{cases} \qquad i = 1, \dots, m,\ j = 1, \dots, n$$

Projecting the result image towards θ-axis, a $1 \times m$ vector $\mathrm{lpt}(t) = (a_1, a_2, \dots, a_i, \dots, a_m)$ is obtained. $a_i (i = 1, \dots, m)$ is defined as follows:

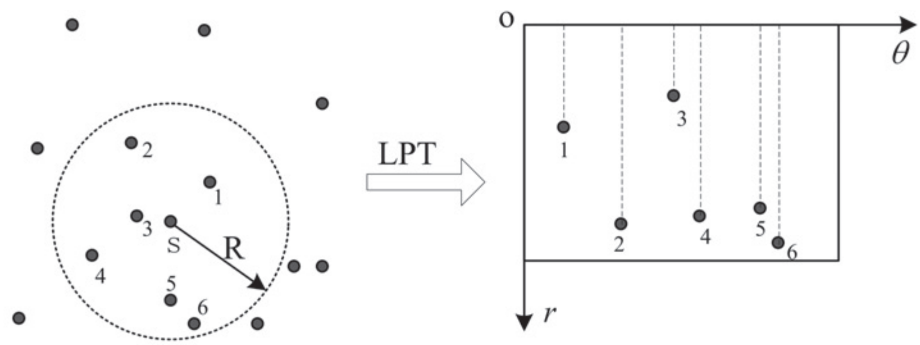
1) If for each $j \in (1, \dots, n)$, $A(i, j) = 0$, then $a_i = 0$;
2) If $j \in (1, \dots, n)$ that makes $A(i, j) = 1$ exists, then a_i is equal to the minimum of j that makes $A(i, j) = 1$.

The vector $\mathrm{lpt}(t)$ is the pattern of measured star t through LPT. For onboard guide stars, patterns can be generated in a similar way. Usually, vector $\mathrm{lpt}(t)$ consists of a lot of zero bits and few nonzero bits which are corresponding to the neighboring stars. It is clear that zero bits represent the spacing among neighboring stars in circular direction

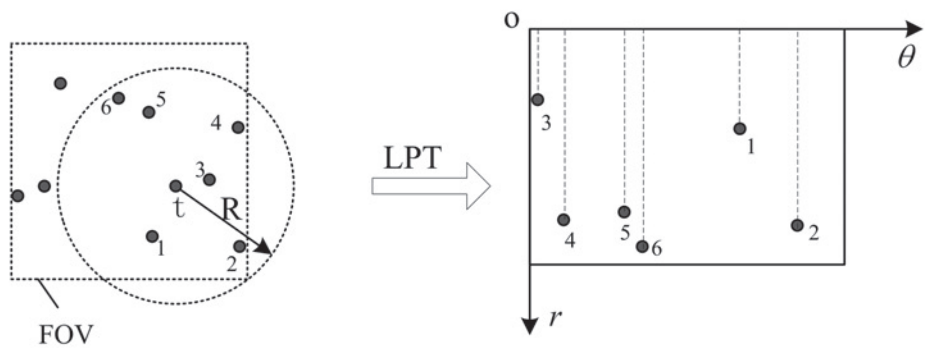


(a) Binary image **(b) Result image through LPT**

Fig. 1 Log-Polar transform.



(a) Star image of a part of guide star catalog and its result of LPT



(b) Measured star image and its result of LPT

Fig. 2 LPT of star image.

while nonzero bits represent the distances between neighboring stars and center star (star t) in radial direction. This is how the LPT makes use of the geometric distribution of neighboring stars to generate star patterns.

Denoting $\mathrm{lpt}(t)$ and $\mathrm{lpt}(s)$ as the pattern of star t in measured star image and the pattern of guide star s in guide star catalog, the similarity of these two patterns is defined as

$$\mathrm{sim}(\mathrm{lpt}(s), \mathrm{lpt}(t)) = \max_{v=1}^{m} \ \mathrm{same}(\mathrm{cs}(\mathrm{lpt}(s), v), \mathrm{lpt}(t)) \tag{2}$$

Here $\mathrm{cs}(\mathrm{lpt}(s), v)$ means circular shifting $\mathrm{lpt}(s)$ for v bits, and "same" is defined as the number of matched nonzero bits. The bigger the same value, the more similar these two vectors are. So $\mathrm{sim}(\mathrm{lpt}(s), \mathrm{lpt}(t))$ represents the similarity between $\mathrm{lpt}(t)$ and $\mathrm{lpt}(s)$. If measured star t matches guide star s, their patterns meet the condition

$$\mathrm{sim}(\mathrm{lpt}(s), \mathrm{lpt}(t)) > \xi \tag{3}$$

Here ξ is a similarity threshold which is determined by the number of nonzero bits in pattern vector (number of neighbor stars).

III. String Coding and Recognition

As mentioned above, star pattern is a $1 \times m$ vector with few nonzero bits. A lot of extra memory is needed to store the patterns if these vectors are directly used for recognition. Moreover, the search progress in recognition is time wasting and very inefficient. To avoid this kind of disadvantage, pattern vector of star is first coded into string and then recognized by the means of string match. The approach of coding is as follows:

1) Circular shift all zero bits ahead of the first nonzero bit in $\mathrm{lpt}(t)$ to the tail and thus the nonzero bit becomes the first bit of $1 \times m$ vector.

2) Code the circular shifted vector into a string: Each character in the coded string is stored by 1 byte. The odd bytes of string correspond to the nonzero bits of vector and the even bytes of string represent the numbers of zero bits between two adjacent nonzero bits. For example, LPT result ($\mathrm{lpt}(s)$) of guide star s in guide star catalog and its pattern string ($\mathrm{str}(s)$) are shown below:

$$\mathrm{lpt}(s) = 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 35\ 0\ 0\ 0\ 0\ 0\ 0\ 53\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 44\ 0\ 0\ 0\ 0\ 0\ 52\ 51\ 0\ 0\ 0\ 0\ 0\ 0\ 0$$
$$54\ 48\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 31\ 0\ 49\ 0\ 0\ 00\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 53\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$$
$$29\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$$

$$\mathrm{str}(s) = 35\ 6\ 53\ 9\ 44\ 5\ 52\ 0\ 51\ 7\ 54\ 0\ 48\ 10\ 31\ 1\ 49\ 14\ 53\ 16\ 29\ 21$$

Based on the fact that rotation is transformed into circular shift through LPT, the pattern strings of guide stars are extended by duplicating the whole string itself and appending it to the tail. Thus simple shift instead of complicated circular shift is needed and quick matching is accomplished. It is noticeable that the pattern strings of measured stars have no changes. For example, the pattern string of guide star s is expanded to

$$\mathrm{str}'(s) = 35\ 6\ 53\ 9\ 44\ 5\ 52\ 0\ 51\ 7\ 54\ 0\ 48\ 10\ 31\ 1\ 49\ 14\ 53\ 16\ 29\ 21\ 35\ 6\ 53\ 9\ 44\ 5\ 52\ 0\ 51\ 7\ 54$$
$$0\ 48\ 10\ 31\ 1\ 49\ 14\ 53\ 16\ 29\ 21$$

After LPT transform and string coding, the pattern of a star can be regarded as a word that consists of some letters or characters. Correspondingly, the set including the pattern strings of all guide stars can be regarded as a dictionary. Thus, the process of star identification is identical with finding the most similar word in a dictionary.

Given a pattern string (word) $\mathrm{str}_{1\times m}$ and a dictionary $\mathrm{dict} = \{p_1, p_2, \ldots, p_N\}$, lookup is described as finding a pattern string p_i in dict which matches $\mathrm{str}_{1\times m}$. If all the words in the dictionary are combined into a single long sentence $\mathrm{tex}_{1\times k}$, the process of lookup becomes a problem of string match to finding the correct position where $\mathrm{str}_{1\times m}$ appears in $\mathrm{tex}_{1\times k}$. Here $\mathrm{str}(i) \in \Re$, $i = 1, 2, \ldots, m$, $\mathrm{tex}(i) \in \Re$, $i = 1, 2, \ldots, k$, $\Re$ is denoted as character set and generally $k \gg m > 0$.

Generally, the rules of character match in string match strategy are known as simple as $a = a$, $b = b$. But these rules of character match do not apply to star identification because the exact meaning of character for star pattern is different from the general case. In some sense, the character match for star identification is matching of odd bits. Moreover, the string match for star identification should be a kind of approximate string match approach to guarantee the robustness of algorithm. Hence, the match rules of string match for star identification must be redefined. Assuming a_{2i+1} and b_{2i+1} are characters in pattern strings of measured star and guide star, respectively, while a_1 and b_1 are start points where the match of two string begins (as shown in Fig. 3). If a_{2i+1} matches b_{2i+1}, then the following condition must be satisfied.

$$|a_{2i+1} - b_{2i+1}| \leqslant 1 \quad \text{and} \quad |(a_2 + a_4 + \cdots + a_{2i}) - (b_2 + b_4 + \cdots + b_{2i})| \leqslant 1 \tag{4}$$

The complexity of traverse method for string match is $m(k - m - 1)$, which means $m(k - m - 1)$ times comparing is needed in the worst case. As a kind of popular solution for quick string match, the Knuth–Morris-Pratt (KMP) algorithm [14] reduces the complexity to $m + k$ by making full use of past comparing information and designing a failure link array.

Base on the KMP algorithm, an approximate string match algorithm using the redefined character match rules suitable for the recognition of star pattern string is introduced. Unlike general approximate string match algorithms

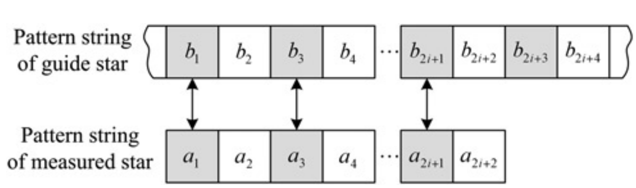


Fig. 3 Character match redefinition.

which must deal with operations as deleting, inserting, and substituting, the matching strategy for star pattern string is comparatively easier.

The number of matches and mismatches between the corresponding characters of measured star and guide star are recorded and updated, respectively, in the matching processing. The matching flow is simply controlled by tracking these two numbers. Denoting n_{match} and n_{mismatch} as number of matches and mismatches, the string match for star identification follows principles as follows:

1) *Correct match condition*: If the number of matched characters n_{match} exceeds a certain threshold ξ_1, a correct match is found; ξ_1 is determined by the number of neighboring stars (n_{neighbor}) to form a star pattern. In experiment, ξ_1 is set to $n_{\mathrm{neighbor}} - 2$, which means two characters mismatches are allowed.
2) *Wrong match condition*: When the number of mismatched characters n_{mismatch} exceeds a certain threshold ξ_2 (ξ_2 is set to 2 in experiment), it means that no correct matches will be found for the current selected guide star. By this means, the recognition process is speeded up by quickly jumping over the wrong matches. In case of wrong match, the return status of string match is determined by the failure link array of KMP algorithm.
3) *Searching range limitation*: In case of correct match, the number of neighboring stars of measured star and guide star will match each other roughly, so the number of neighboring stars of guide star (m_{neighbor}) should lie in a certain range determined by the number of neighbor stars of measured star (n_{neighbor}). Usually, m_{neighbor} is slightly larger (or equal) than n_{neighbor} due to square FOV. If $m_{\mathrm{neighbor}} < n_{\mathrm{neighbor}} - 2$ or $m_{\mathrm{neighbor}} > 2n_{\mathrm{neighbor}}$, this guide star is quickly skipped and the next guide star is selected for checking. The limitation of search range also greatly speeds up the process of string match.

IV. Simulations and Result Analysis

To evaluate the performance of the proposed algorithm, star image is simulated to verify star identification. The parameters of imaging system for star image simulation are listed as follows: The FOV is $12^\circ \times 12^\circ$ with image size of 1024×1024. The pixel size is $12\,\mu\mathrm{m} \times 12\,\mu\mathrm{m}$ and the main point (where optical axis crosses the image plane) is (512, 512). The focal length of optical lens is 58.4563 mm and maximum sensitivity of star magnitude is 6 Mv. The simulations are performed by following steps:

Step 1: Select guide stars brighter than 6 Mv from the standard star catalog. The Smithsonian Astrophysical Observatory 2000 star catalog is used in this paper.

Step 2: Based on the imaging model of star sensor, generate a simulated star image using a random predetermined attitude and the imaging system parameters above.

Step 3: Star identification algorithm is carried out to identify the star map generated in step 2.

Step 4: Repeat steps 2 and 3 1000 times and record the identification results. The statistics comes from the recorded identification results.

Because stars near the border of FOV have incomplete star pattern due to square FOV, the risk of identification failure arises. It can be easily concluded that those stars near the center of FOV (with larger n_{neighbor}) are most likely recognized correctly. Usually, the stars near the center of FOV have large number of neighbor stars which lead to high probability of correct identification, so the stars with larger n_{neighbor} have priority to be selected first. Moreover, brighter stars are usually regarded as reliable stars for star identification and also have priority. On the basis of comprehensive consideration of these two facts, brighter stars with larger n_{neighbor} have priority to be chosen for matching. Accordingly, a special value $Q = M/n_{\mathrm{neighbor}}$ (M as magnitude) is defined to rank the measured stars in FOV. After quick sort, those stars with least Q have priority to be selected first. Figure 4 is the whole flow chart of star identification algorithm based on LPT.

Since pattern radius R is a very important parameter for the proposed algorithm, a simple experiment on how to determinate R is done first. The identification rate under different R is shown in Fig. 5. It can be seen from the figure that the highest identification rate is obtained when pattern radius R is around 6° which is equal to half size of FOV, so R is set to 6° in the simulations.

Grid algorithm is a typical and excellent star pattern-based algorithm with good performance in robustness, memory and time usage [9,10]. In order to comprehensively evaluate our algorithm, comparison of these two algorithms under the same condition is carried out. The pattern radius R_p of grid algorithm is set to 6°, and the grid parameter $g^2 = 60 \times 60$. For both algorithms, the result is obtained from the statistics of identification of 1000 random simulated star images around the celestial sphere.

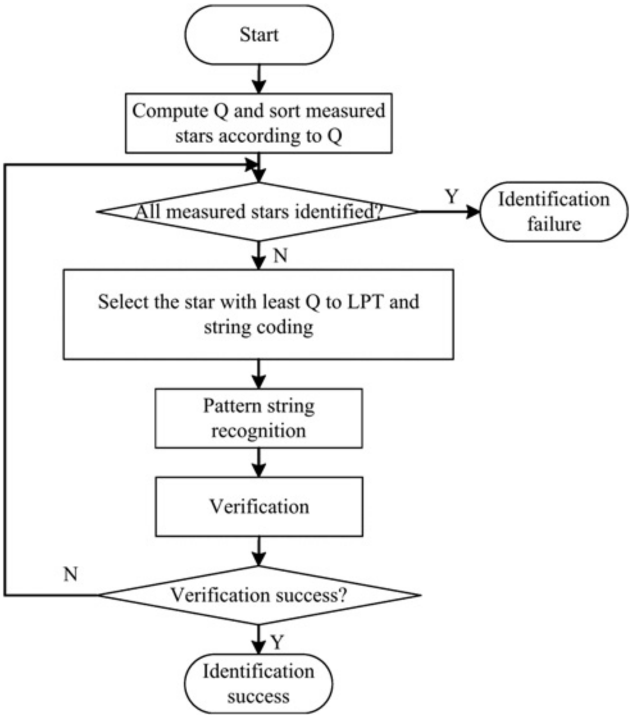


Fig. 4 Algorithm flow chart.

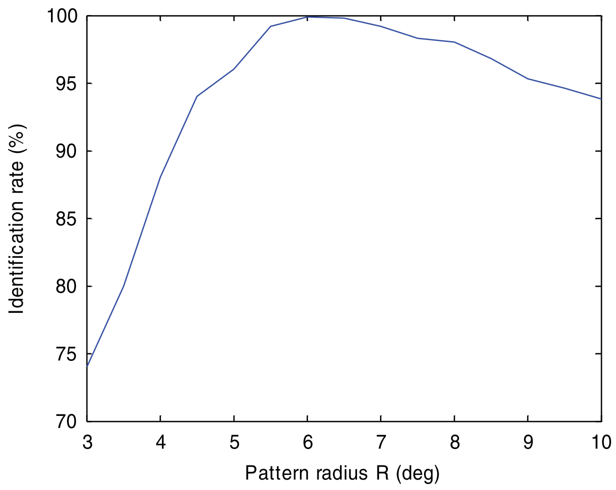


Fig. 5 Identification rate under different *R*.

Since correct star identification is crucial for the initial attitude establishment of a star sensor, the capability of reliability and robustness is the most important characteristic for a star identification algorithms. The robustness against positional noise and magnitude (brightness) noise is mainly investigated in this paper. The identification rate under different positional noise levels is shown in Fig. 6a. It is obvious that the identification rate of our algorithm is improved compared with grid algorithm, especially under the high-positional locating noise level. The identification rate of our algorithm remains above 98% when the standard deviation of positional locating noise is two pixels, while the identification rate of grid algorithm drops to about 94%. The identification rate under different magnitude

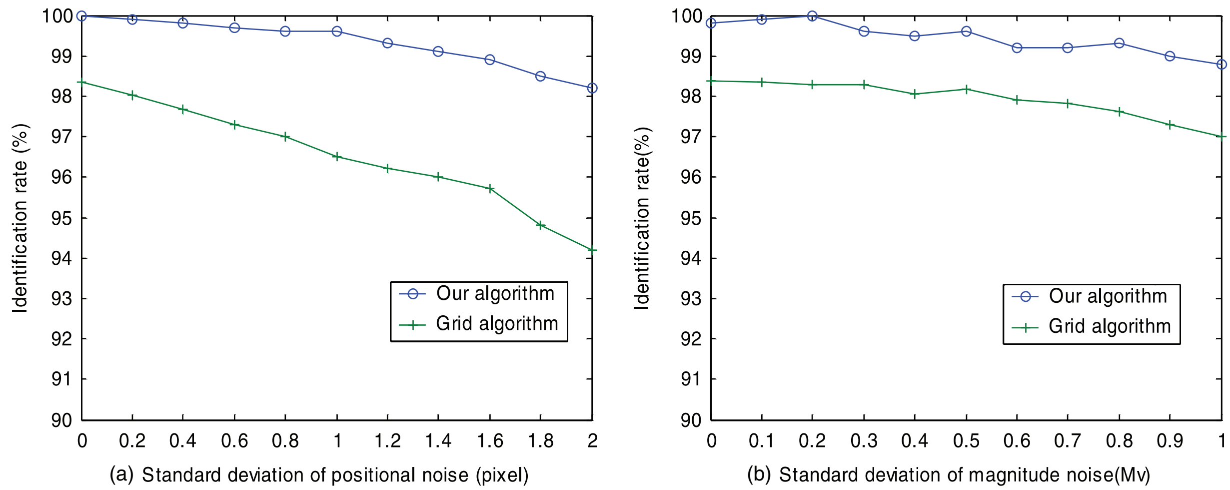


Fig. 6 Identification rate against positional noise and magnitude noise.

noise levels is shown in Fig. 6b. It can be seen that both algorithms are robust against magnitude noise, while our algorithm gains the advantage over grid algorithm all along.

Both algorithms are implemented in C language and run on an Intel PIII-800 computer. The average length of pattern string is 28, so the total storage requirement of our algorithm is 94 Kbytes if one character is stored by one byte. Under the same condition, the storage requirement of grid algorithm is 144 Kbytes. The average run time of our algorithm is 15.7 ms while grid algorithm is 10.5 ms, which is relatively shorter. The main reason for more time usage is that string match is more time-costly. Especially when there are a huge number of stars in FOV the identification time increases remarkably. However, the run time of our algorithm is still very short compared with conventional star identification algorithms such as triangle algorithm, polygon match algorithm, etc. [10].

V. Conclusion

As a useful means for scale, shift, and rotation invariant target recognition, LPT is introduced for star identification and a Log-Polar-based star identification algorithm is proposed accordingly. Star patterns are generated by LPT and then coded to strings. A modified string match algorithm based on KMP is used to find the matches. In simulations, the algorithm shows good performance in robustness against positional and magnitude noise which meets the demand of full sky autonomous star identification without any prior attitude. Meanwhile, the storage requirement is comparatively small and the time usage is acceptable.

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